Semiclassical theory of spin-polarized shot noise in mesoscopic diffusive conductors

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We study fluctuations of spin-polarized currents in a three-terminal spin-valve system consisting of a diffusive normal metal wire connected by tunnel junctions to three ferromagnetic terminals. Based on a spin-dependent Boltzmann-Langevin equation, we develop a semiclassical theory of charge and spin currents and the correlations of the currents fluctuations. In the three terminal system, we show that current fluctuations are strongly affected by the spin-flip scattering in the normal metal and the spin polarizations of the terminals, which may point in different directions. We analyze the dependence of the shot noise and the cross-correlations on the spin-flip scattering rate in the full range of the spin polarizations and for different magnetic configurations. Our result demonstrate that noise measurements in multi-terminal devices allow to determine the spin-flip scattering rate by changing the polarizations of ferromagnetic terminals.

I. INTRODUCTION

Spin-dependent electronic transport and use of the spin degree of freedom of electrons in hybrid magnetic structures have recently been subject of highly interesting field termed spintronics. This developing field has emerged from many exciting phenomena, the giant magneto-resistance being the most well known example, which has made it attractive for applications as well as fundamental studies[1,2,3,4,5,6,7,8]. Typical studied systems are based on the use of ferromagnetic metals (F) and/or external magnetic fields to inject, manipulate and detect spin-polarized electrons inside a mesoscopic normal metal (N).

An important characteristic of mesoscopic systems is the appearance of shot noise, the fluctuations of current through the system due to the randomness of the electronic scattering processes and the quantum statistics. Shot noise and nonlocal correlations of the current fluctuations contain additional information on the conduction process which is not gainable through a mean current measurement. In ferromagnetic-normal metal structures, in which the spin of electrons plays an essential rule, current fluctuations are due to the randomness of both charge and spin transport processes. Thus, shot noise measurements are expected to provide information about spin-dependent scattering processes and spin accumulation in the system. Together with the importance of noise in spintronic devices in view of applications this motivates our study of spin-polarized shot noise.

In the past years, shot noise has been extensively studied in a wide variety of hybrid structures involving normal metals, semiconductors and superconductors[9,10]. However, there are few studies devoted to shot noise in ferromagnet-normal metal systems. Results of earlier studies of current fluctuations in FNF double barrier systems in the Coulomb blockade regime[11] and FIF (I being an insulator) systems[12] can be understood in terms of well known results for the corresponding normal-metal systems for two spin directions[9]. Tserkovnyak and Brataas studied shot noise in double barrier FNF systems with noncollinear magnetizations in F-terminals[13]. They found that the shot noise has a non-monotonic behavior with respect to the relative angle of the magnetizations for different scattering regimes and different types of FN junctions.

The effect of spin flip scattering on spin-polarized current fluctuations has been considered in Refs. 14,15,16,17,18. Mishchenko[14] found that in a perfectly polarized two-terminal double barrier system spin-flip scattering leads to a strong dependence of shot noise on the relative orientation of the magnetizations in F-terminals. In Ref. 18 we have proposed a four-terminal spin-valve system of tunnel junctions to study spin-dependent shot noise and cross-correlations simultaneously. It has been found that the cross-correlations between currents in terminals with opposite spin polarization can be used to measure directly the spin-flip scattering rate.

Recently, there have been also studies of the current fluctuations of spin-polarized entangled electrons in quantum dots and wires[19] and in quantum dots attached to the ferromagnetic leads in the Coulomb blockade[20], Kondo[21] and sequential tunneling[22,23,24,25,26] regimes.

In this paper, we study current fluctuations in a three terminal diffusive FNF system in the full range of spin polarizations and the spin-flip scattering intensity. Based on the Boltzmann-Langevin[27,28,29,30] kinetic approach, we develop a semiclassical theory for spin-polarized transport in the presence of the spin-flip scattering. We obtain the basic equations of charge and spin transport, which allow the calculation of mean currents and the correlations of current fluctuations in multi-terminal diffusive systems. Applying the developed formalism to a three-terminal geometry, we find that current correlations are affected strongly by spin-flip scattering and spin polarizations. We focus on the shot noise of the total current through the system and the cross-correlations measured between currents of two terminals. We demonstrate how these correlations deviate from the noise characteristics of the unpolarized system, depending on the spin-flip scattering rate, the polarizations of the terminals and their magnetic configurations (relative directions). Our results provide a full analysis for spin-dependent shot noise and cross-correlations in terms of the relevant parameters.

The structure of the paper is as follows. In section II we extend the Boltzmann-Langevin equations to the diffusive systems, in which spin-flip scattering takes place and which are connected to ferromagnetic terminals. We find the basic equa-

tions of the charge and the spin currents and the correlations of their fluctuations. In section III we apply this formalism to the three terminal system. We obtain the Fano factor and the cross-correlations between currents through two different terminals. Section VI is devoted to the analysis of the calculated quantities. We present analytical expressions for the Fano factor and the cross-correlations in different important limits. Finally, we end with some conclusion in section V.

II. BOLTZMANN-LANGEVIN EQUATIONS WITH SPIN-FLIP SCATTERING

In this section we extend the semiclassical Boltzmann-Langevin kinetic approach[9] to cover spin-polarized transport. In the presence of spin-flip scattering the Boltzmann-Langevin equation is written as

$$\frac{d}{dt}f_\alpha = I^{\rm imp}[f_\alpha] + I^{\rm sf}_\alpha[f_\alpha, f_{-\alpha}] + \xi^{\rm imp}_\alpha + \xi^{\rm sf}_\alpha, \quad (1)$$

where the fluctuating distribution function of spin $\alpha(=\pm1)$ electrons $f_\alpha(\mathbf{p},\mathbf{r},t) = \bar{f}_\alpha(\mathbf{p},\mathbf{r}) + \delta f_\alpha(\mathbf{p},\mathbf{r},t)$ depends on the momentum $\mathbf{p}$, the position $\mathbf{r}$, and the time t. Eq. (1) contains both normal impurity and spin-flip collision integrals which are given by the relations

$$I^{\rm imp}[f_\alpha] = \Omega \int \frac{d\mathbf{p}'}{(2\pi h)^3}[J_{\alpha\alpha}(\mathbf{p}',\mathbf{p}) - J_{\alpha\alpha}(\mathbf{p},\mathbf{p}')], \quad (2)$$

$$I^{\rm sf}_\alpha[f_\alpha, f_{-\alpha}] = \Omega \int \frac{d\mathbf{p}'}{(2\pi h)^3}[J_{-\alpha\alpha}(\mathbf{p}',\mathbf{p}) - J_{\alpha-\alpha}(\mathbf{p},\mathbf{p}')]. \quad (3)$$

Here $J_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r},t) = W_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r})f_\alpha(\mathbf{p},\mathbf{r},t)[1 - f_{\alpha'}(\mathbf{p}',\mathbf{r},t)]$, where $W_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r})$ is the elastic scattering rate from the state $\mathbf{p},\alpha$ into $\mathbf{p}',\alpha'$; Ω is the volume of the system. The corresponding Langevin sources of fluctuations due to the random character of the electron scattering are given by

$$\xi^{\rm imp}_\alpha = \Omega \int \frac{d\mathbf{p}'}{(2\pi h)^3}[\delta J_{\alpha\alpha}(\mathbf{p}',\mathbf{p}) - \delta J_{\alpha\alpha}(\mathbf{p},\mathbf{p}')], \quad (4)$$

$$\xi^{\rm sf}_\alpha = \Omega \int \frac{d\mathbf{p}'}{(2\pi h)^3}[\delta J_{-\alpha\alpha}(\mathbf{p}',\mathbf{p}) - \delta J_{\alpha-\alpha}(\mathbf{p},\mathbf{p}')], (5)$$

where the random variable $\delta J_{\alpha\alpha'}(\mathbf{p}',\mathbf{p},\mathbf{r},t)$ is the fluctuation of the current $J_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r},t) = \bar{J}_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r},t) + \delta J_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r},t)$; with $\bar{J}_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r},t) = W_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r})\bar{f}_\alpha(\mathbf{p},\mathbf{r},t)[1 - \bar{f}_{\alpha'}(\mathbf{p}',\mathbf{r},t)]$ being the mean current.

We will assume that all scattering events are independent elementary processes and thus the correlator of the current fluctuations $\delta J_{\alpha\alpha'}(\mathbf{p},\mathbf{p}',\mathbf{r},t)$ is that of a Poissonian process:

$$<\delta J_{\alpha_1\alpha_2}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r},t)\delta J_{\alpha_3\alpha_4}(\mathbf{p}_3,\mathbf{p}_4,\mathbf{r}',t')>= \frac{(2\pi h)^6}{\Omega}\delta_{\alpha_1\alpha_3}\delta_{\alpha_2\alpha_4}\delta(\mathbf{p}_1-\mathbf{p}_3)\delta(\mathbf{p}_2-\mathbf{p}_4) \times\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')\bar{J}_{\alpha_1\alpha_2}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r},t). \quad (6)$$

Due to the non-vanishing spin-flip collision integral in (1), the equations for the distributions of electrons with opposite spin directions are coupled. The coupled equations can be transformed into two decoupled equations for the charge $f_{\rm c} = \sum_\alpha f_\alpha/2$ and spin $f_{\rm s} = \sum_\alpha \alpha f_\alpha/2$ distribution functions, which read

$$\frac{d}{dt}f_{\rm c(s)} = I^{\rm imp}[f_{\rm c(s)}] + I^{\rm sf}_{\rm c(s)}[f_{\rm c(s)}] + \xi^{\rm imp}_{\rm c(s)} + \xi^{\rm sf}_{\rm c(s)}. \quad (7)$$

Here we have introduced different collision integrals:

$$I^{\rm imp}[f_{\rm c(s)}] = \Omega \int \frac{d\mathbf{p}'}{(2\pi h)^3}[J^{\rm imp}_{\rm c(s)}(\mathbf{p}',\mathbf{p}) - J^{\rm imp}_{\rm c(s)}(\mathbf{p},\mathbf{p}')], \quad (8)$$

$$I^{\rm sf}_{\rm c}[f_{\rm c}] = \Omega \int \frac{d\mathbf{p}'}{(2\pi h)^3}[J^{\rm sf}_{\rm c}(\mathbf{p}',\mathbf{p}) - J^{\rm sf}_{\rm c}(\mathbf{p},\mathbf{p}')], \quad (9)$$

$$I^{\rm sf}_{\rm s}[f_{\rm s}] = -\Omega \int \frac{d\mathbf{p}'}{(2\pi h)^3}W^{\rm sf}(\mathbf{p},\mathbf{p}')[f_{\rm s}(\mathbf{p}') + f_{\rm s}(\mathbf{p})], \quad (10)$$

where

$$J^{\rm imp(sf)}_{\rm c(s)}(\mathbf{p},\mathbf{p}',\mathbf{r},t) = W^{\rm imp(sf)}(\mathbf{p},\mathbf{p}',\mathbf{r}) \times f_{\rm c(s)}(\mathbf{p},\mathbf{r},t)[1 - f_{\rm c(s)}(\mathbf{p}',\mathbf{r},t)], \quad (11)$$

and we assumed $W_{\alpha\alpha'}(\mathbf{p},\mathbf{p}') = W_{\alpha'\alpha}(\mathbf{p}',\mathbf{p})$, $W_{+-} = W_{-+} = W^{\rm sf}$; $W_{++} = W_{--} = W^{\rm imp}$. The corresponding Langevin sources of fluctuations are given by

$$\xi^{\rm imp(sf)}_{\rm c}(\mathbf{p},\mathbf{r},t) = \frac{1}{2}\sum_\alpha \xi^{\rm imp(sf)}_\alpha(\mathbf{p},\mathbf{r},t), \quad (12)$$

$$\xi^{\rm imp(sf)}_{\rm s}(\mathbf{p},\mathbf{r},t) = \frac{1}{2}\sum_\alpha \alpha\xi^{\rm imp(sf)}_\alpha(\mathbf{p},\mathbf{r},t). \quad (13)$$

In the following we assume that all the quantities are sharply peaked around the Fermi energy and instead of $\mathbf{p}$ use the quantities ε the energy and $\mathbf{n}$ the direction of the Fermi momentum. Then, for elastic scattering of electrons the following relation holds

$$\Omega W^{\rm imp(sf)}(\mathbf{p},\mathbf{p}',\mathbf{r}) = \frac{2}{N_0}\delta(\varepsilon-\varepsilon')w^{\rm imp(sf)}(\mathbf{n},\mathbf{n}',\mathbf{r}), \quad (14)$$

where N_0 is the density of states in the Fermi level.

For a diffusive conductor we apply the standard diffusive approximation to the kinetic equations (7) where the charge and spin distribution functions are split into the symmetric and asymmetric parts:

$$f_{\rm c(s)}(\mathbf{n},\varepsilon,\mathbf{r},t) = f_{\rm c(s)0}(\varepsilon,\mathbf{r},t) + \mathbf{n}.\mathbf{f}_{\rm c(s)1}(\varepsilon,\mathbf{r},t). \quad (15)$$

Substituting this form of $f_{\rm c}$ in Eqs. (7) and averaging subsequently over the Fermi momentum direction first weighted with one and then with $\mathbf{n}$, we obtain

$$\frac{v_F}{3}\nabla.\mathbf{f}_{\rm c1} = \int d\mathbf{n}\xi^{\rm sf}_{\rm c}(\mathbf{n},\varepsilon,\mathbf{r},t), \quad (16)$$

$$\frac{v_F}{3}\nabla f_{\rm c0} = -\frac{1}{3\tau}\mathbf{f}_{\rm c1} + \int \mathbf{n}d\mathbf{n}[\xi^{\rm imp}_{\rm c}(\mathbf{n},\varepsilon,\mathbf{r},t) + \xi^{\rm sf}_{\rm c}(\mathbf{n},\varepsilon,\mathbf{r},t)]. \quad (17)$$

In the same way from Eqs. (15) and (7), for f_s, we obtain

$$\frac{v_F}{3}\nabla.\mathbf{f}_{s1} = -\frac{1}{\tau_0}f_{s0} + \int d\mathbf{n}\xi_s^{sf}(\mathbf{n},\varepsilon,\mathbf{r},t), \qquad (18)$$

$$\frac{v_F}{3}\nabla f_{s0} = -\frac{\mathbf{f}_{s1}}{3\tau_s}$$

$$+ \int \mathbf{n}d\mathbf{n}[\xi_s^{imp}(\mathbf{n},\varepsilon,\mathbf{r},t) + \xi_s^{sf}(\mathbf{n},\varepsilon,\mathbf{r},t)], \qquad (19)$$

where different relaxation times of normal impurity and spin-flip scatterings are defined as

$$\frac{1}{\tau_{c(s)}} = \frac{1}{\tau_{imp}} + \frac{1}{\tau_{sf}^{-(+)}}, \qquad (20)$$

$$\frac{1}{\tau_{sf}} = \frac{1}{\tau_{sf}^-} + \frac{1}{\tau_{sf}^+}, \qquad (21)$$

$$\frac{\mathbf{n}}{\tau_{imp}} = \int d\mathbf{n}'w^{imp}(\mathbf{n},\mathbf{n}',\mathbf{r})(\mathbf{n}-\mathbf{n}'), \qquad (22)$$

$$\frac{\mathbf{n}}{\tau_{sf}^\pm} = \int d\mathbf{n}'w^{sf}(\mathbf{n},\mathbf{n}',\mathbf{r})(\mathbf{n}\pm\mathbf{n}'). \qquad (23)$$

Here we used the identity $w^{imp(sf)}(\mathbf{n},\mathbf{n}',\mathbf{r}) = w^{imp(sf)}(|\mathbf{n}-\mathbf{n}'|,\mathbf{r})$, for the elastic scattering.

In obtaining Eqs. (16-19) we have disregarded terms $(\partial/\partial t)f_{c(s)}$ in the expressions $(d/dt)f_{c(s)} = (\partial/\partial t + v_F\mathbf{n}.\nabla + e\mathbf{E}.\nabla_{\mathbf{p}})f_{c(s)}$, since we are only interested in the zero frequency noise power. The terms of the electric field $e\mathbf{E}.\nabla_{\mathbf{p}}$ are eliminated by substituting ε by $\varepsilon - e\varphi_{c(s)}(\mathbf{r},t)$ in the arguments of $f_{c(s)}$, respectively, where the charge and spin potentials are expressed as $\varphi_c = \sum_\alpha \varphi_\alpha/2$ and $\varphi_s = \sum_\alpha \alpha\varphi_\alpha/2$, with $\varphi_\alpha(\mathbf{r},t) = \int d\varepsilon f_{\alpha 0}(\varepsilon,\mathbf{r},t)$, being the spin-dependent electro-chemical potential. We also used the identities $\int d\mathbf{n}\xi_{c(s)}^{imp} = 0$, which follows from the conservation of number of spin α electrons in each normal impurity scattering process[9]. In contrast to this, we note that the integral $\int d\mathbf{n}\xi_{c(s)}^{sf}$ does not vanish, reflecting the fact that spin is not conserved by the spin-flip process.

Combining Eqs. (16), (17) and (18), (19) the equations for the symmetric parts of the mean charge and spin distribution functions are obtained as

$$\nabla^2\bar{f}_{c0} = 0, \qquad (24)$$

$$\nabla^2\bar{f}_{s0} = \frac{\bar{f}_{s0}}{\ell_{sf}^2}, \qquad (25)$$

where $\ell_{sf} = \sqrt{D_s\tau_{sf}}$ is the spin-flip length. Note, that in general charge and spin diffusion constants given by $D_{c(s)} = v_F^2\tau_{c(s)}/3$ are different.

Using Eqs. (15) the charge and spin current densities can be expressed as $\mathbf{j}_{c(s)} = (eN_0v_F/3)\int d\varepsilon\mathbf{f}_{c(s)1}$. The corresponding fluctuating potentials are given by $\bar{\varphi}_{c(s)}(\mathbf{r}) + \delta\varphi_{c(s)}(\mathbf{r},t) = (1/e)\int d\varepsilon f_{c(s)0}$. Using these identities and integrating Eqs. (16-17) over ε we obtain diffusion equations for the charge

potential and current density,

$$\nabla.\bar{\mathbf{j}}_c = 0, \qquad (26)$$

$$\nabla.\delta\mathbf{j}_c = i_c^{sf} \qquad (27)$$

$$\bar{\mathbf{j}}_c = -\sigma\nabla\bar{\varphi}_c, \qquad (28)$$

$$\delta\mathbf{j}_c = -\sigma\nabla\delta\varphi_c + \mathbf{j}_c^c, \qquad (29)$$

which also imply

$$\nabla^2\bar{\varphi}_c = 0. \qquad (30)$$

In the same way Eqs. (18-19) give us diffusion equations of spin potential and current density:

$$\nabla.\bar{\mathbf{j}}_s = -\frac{e^2\nu_F}{\tau_{sf}}\bar{\varphi}_s, \qquad (31)$$

$$\nabla.\delta\mathbf{j}_s = -\frac{e^2\nu_F}{\tau_{sf}}\delta\varphi_s + i_s^{sf}, \qquad (32)$$

$$\bar{\mathbf{j}}_s = -\sigma_s\nabla\bar{\varphi}_s, \qquad (33)$$

$$\delta\mathbf{j}_s = -\sigma_s\nabla\delta\varphi_s + \mathbf{j}_s^c, \qquad (34)$$

$$\nabla^2\bar{\varphi}_s = \frac{\bar{\varphi}_s}{\ell_{sf}^2}, \qquad (35)$$

where $\sigma_{c(s)} = e^2N_0D_{c(s)}$ are the charge and spin conductivities. Here

$$\mathbf{j}_{c(s)}^c = ev_FN_0\tau_{c(s)}\int(\xi_{c(s)}^{imp} + \xi_{c(s)}^{sf})\mathbf{n}d\mathbf{n}d\varepsilon \qquad (36)$$

are the Langevin sources of fluctuations of the charge and spin current densities, and

$$i_{c(s)}^{sf}(\mathbf{r},t) = eN_0\int d\varepsilon d\mathbf{n}\xi_{c(s)}^{sf}, \qquad (37)$$

are additional terms in the expressions for the divergence of charge and spin currents fluctuations Eqs. (27) and (32), due to the non-conserved nature of spin-flip process.

Now we calculate possible correlations between the currents $\mathbf{j}_{c(s)}^c$ and $i_{c(s)}^{sf}$. From Eqs. (4-6) for the correlations of different fluctuating sources we obtain

$$< \xi_\alpha^{imp(sf)}(\mathbf{n},\varepsilon,\mathbf{r},t)\xi_{\alpha'}^{imp(sf)}(\mathbf{n}',\varepsilon',\mathbf{r}',t') > = \frac{1}{N_0}$$
$$\times\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')\delta(\varepsilon-\varepsilon')G_{\alpha\alpha'}^{imp(sf)}(\mathbf{n},\mathbf{n}',\mathbf{r},\varepsilon), \quad (38)$$

$$< \xi_\alpha^{imp}(\mathbf{n},\varepsilon,\mathbf{r},t)\xi_{\alpha'}^{sf}(\mathbf{n}',\varepsilon',\mathbf{r}',t') > = 0, \qquad (39)$$

where

$$G_{\alpha\alpha'}^{imp} = \delta_{\alpha\alpha'}\int d\mathbf{n}''[\delta(\mathbf{n}-\mathbf{n}') - \delta(\mathbf{n}'-\mathbf{n}'')]$$
$$[\bar{J}_{\alpha\alpha}(\mathbf{n},\mathbf{n}'',\varepsilon) + \bar{J}_{\alpha\alpha}(\mathbf{n}'',\mathbf{n},\varepsilon)], \qquad (40)$$

$$G_{\alpha\alpha'}^{sf} = \int d\mathbf{n}''[\delta_{\alpha\alpha'}\delta(\mathbf{n}-\mathbf{n}') - \delta_{\alpha-\alpha'}\delta(\mathbf{n}'-\mathbf{n}'')]$$
$$[\bar{J}_{-\alpha\alpha}(\mathbf{n},\mathbf{n}'') + \bar{J}_{\alpha-\alpha}(\mathbf{n}'',\mathbf{n})]. \qquad (41)$$

FIG. 1: A schematic picture of the three terminal spin valve. A diffusive wire of length L is connected to three ferromagnetic terminals via the tunnel junctions.

From Eqs. (12), (13) and (38-41) we calculate the correlations between the fluctuating sources $\xi_{c(s)}^{imp(sf)}$, which can be used together with Eqs. (36) and (37) to obtain the results,

$$< j_{c(s)l}^c(\mathbf{r}, t) j_{c(s)m}^c(\mathbf{r}', t') > = \delta_{lm} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \sigma_{c(s)}$$

$$\times \frac{\tau_{c(s)}}{\tau_{imp}} \sum_\alpha [\Pi_{\alpha\alpha}(\mathbf{r}) + \frac{\tau_{imp}}{\tau_{sf}^{-(+)}} \Pi_{\alpha-\alpha}(\mathbf{r})], \quad (42)$$

$$< j_{cl}^c(\mathbf{r}, t) j_{sm}^c(\mathbf{r}', t') > = \delta_{lm} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \sigma_c$$

$$\times \sum_\alpha \alpha \Pi_{\alpha\alpha}(\mathbf{r}), \quad (43)$$

$$< i_c^{sf}(\mathbf{r}, t) i_{c(s)}^{sf}(\mathbf{r}', t') >$$

$$= < j_{c(s)m}^c(\mathbf{r}', t') i_{c(s)}^{sf}(\mathbf{r}', t') > = 0, \quad (44)$$

$$< i_s^{sf}(\mathbf{r}, t) i_s^{sf}(\mathbf{r}', t') > = \delta_{lm} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \sigma_c \quad (45)$$

$$\times \frac{1}{D_c \tau_{sf}^+} \sum_\alpha \Pi_{\alpha-\alpha}(\mathbf{r}), \quad (46)$$

where

$$\Pi_{\alpha\alpha'}(\mathbf{r}) = \int d\varepsilon \bar{f}_{\alpha 0}(\varepsilon, \mathbf{r}) [1 - \bar{f}_{\alpha' 0}(\varepsilon, \mathbf{r})]. \quad (47)$$

The diffusion equations (24-35) and Eqs. (42-47) are a complete set of equations, which in principal can be solved for a multi-terminal mesoscopic diffusive conductor connected to an arbitrary number N of metallic and/or ferromagnetic terminals held at constant potentials. The distribution function $f_n(\varepsilon - eV_n)$ of the terminal $n(= 1, ..., N)$ biased at the voltage V_n determines the boundary conditions of the diffusive equations. In the case where the terminal n is connected by a tunnel junction to the diffusive conductor at the point $\mathbf{r}_n$, the ferromagnetic character of the terminal can be modeled by a spin-dependent conductance $g_{n\alpha}$. The fluctuating spin α current through the junction is given by $I_{n\alpha}(t) = g_{n\alpha} \int d\varepsilon [f_{\alpha 0}(\varepsilon, \mathbf{r}_n, t) - f_n(\varepsilon - eV_n)]$. As the boundary condition this current should match to the value calculated from the diffusive equations $I_{n\alpha} = \int_{A_n} d\mathbf{S} \cdot \mathbf{j}_\alpha(\mathbf{r}_n, t)$, where A_n is the junction area. Here $f_{\alpha 0} = f_{c0} + \alpha f_{s0}$ and $\mathbf{j}_\alpha = \mathbf{j}_c + \alpha \mathbf{j}_s$ are the spin α symmetric part of the distribution function and current density respectively. From the solutions of the diffusion equations the mean charge and spin currents and the correlations of the corresponding fluctuations can be obtained.

Eqs. (24-35) and (42-47) are valid for an arbitrary τ_{imp}/τ_{sf} in the diffusive limit. In the following we will consider the more realistic case of $\tau_{imp} \ll \tau_{sf}$, where the effect of spin-flip scattering on the conductivity of the diffusive metal is neglected. In this case, $\tau_s = \tau_c = \tau_{imp}$ and hence $\sigma_s = \sigma_c = \sigma$. For simplicity, we also assume that the spin-flip scattering is isotropic, i. e., w^{sf} does not depend on the directions $\mathbf{n}$, $\mathbf{n}'$, which implies $\tau_{sf}^- = \tau_{sf}^+ = 2\tau_{sf}$. In the next section we use the above developed formalism to calculate spin-polarized current correlations in a diffusive three-terminal system.

III. THREE-TERMINAL SPIN VALVE

We consider the three-terminal spin valve system as shown in Fig. 1. The system consists of a diffusive normal wire (N) of length L connected by tunnel junctions to three ferromagnetic terminals F_i ($i = 1, 2, 3$). The terminal F_1 is held at the voltage V and the voltage at the terminals $F_{2,3}$ is zero. The tunnel junction i connecting F_i to N has a spin dependent tunneling conductances $g_{i\alpha}$, which equivalently can be characterized by a total conductance $g_i = \sum_\alpha g_{i\alpha}$, and the polarization $p_i = \sum_\alpha \alpha g_{i\alpha}/g_i$. Inside the wire we account for both, normal impurity and spin flip scattering. The length L is much larger than ℓ_{imp} providing a diffusive motion of electrons. The spin-flip length ℓ_{sf} is assumed to be much larger than ℓ_{imp}, but arbitrary as compared to L. We study the influence of the spin-flip scattering on shot noise of the current through the wire and cross-correlations between currents through the terminals F_2 and F_3.

A. Charge and spin currents fluctuations

To start we write the solutions of Eqs. (30) and (35) in terms of the charge (spin) potentials $\varphi_{c(s)}(0)$ and $\varphi_{c(s)}(L)$, at the connecting points $x = 0$ and $x = L$ inside the wire:

$$\bar{\varphi}_{c(s)}(x) = \phi_{c(s)0}(x) \bar{\varphi}_{c(s)}(0) + \phi_{c(s)L}(x) \bar{\varphi}_{c(s)}(L), \quad (48)$$

where the charge and spin potential functions are defined as

$$\phi_{c0}(x) = 1 - \frac{x}{L}, \quad (49)$$

$$\phi_{cL}(x) = \frac{x}{L}, \quad (50)$$

$$\phi_{s0}(x) = \frac{\sinh[\lambda(1 - x/L)]}{\sinh \lambda}, \quad (51)$$

$$\phi_{sL}(x) = \frac{\sinh(\lambda x/L)}{\sinh \lambda}. \quad (52)$$

Here the parameter $\lambda = L/\ell_{sf}$ is the dimensionless measure of the spin-flip scattering inside the N-wire. An expression for the fluctuations of the current through the wire ΔI_{c1} is

obtained if we take the inner product of $\delta\mathbf{j}_c$ in Eq. (29) with $\nabla\phi_{c0}$ and integrate over the volume of the wire:

$$\Delta I_{c1} = -\sigma \int d\mathbf{s} \cdot \nabla\phi_{c0}\delta\varphi_c + \delta I_c^c, \qquad (53)$$

$$\delta I_c^c = \int d\Omega (i_c^{\text{sf}} + \mathbf{j}_c^c \cdot \nabla)\phi_{c0}, \qquad (54)$$

where we also used Eqs. (27) and (30). In a similar way by volume integration of the products $\nabla\phi_{s0(L)} \cdot \delta\mathbf{j}_s$ in Eq. (34), and using Eqs. (32) and (35) we obtain $\Delta I_{s1}(0, L)$, which yields the fluctuations of spin currents at $x = 0$ and $x = L$:

$$\Delta I_{s1}(0, L) = -\sigma \int d\mathbf{s} \cdot \nabla\phi_{s(0,L)}\delta\varphi_s$$
$$+\delta I_s^c(0, L), \qquad (55)$$

$$\delta I_s^c(0, L) = \int d\Omega (i_s^{\text{sf}} + \mathbf{j}_s^c \cdot \nabla)\phi_{s0,L}. \qquad (56)$$

Note, that as a result of the spin-flip scattering the spin current and, hence, its fluctuations are not conserved through the wire. Using Eqs. (55), (56), (51) and (52) a relation between the fluctuation of the spin currents at the two different points is obtained,

$$\Delta I_{s1}(0) = \Delta I_{s1}(L) - g_N\lambda^2 t[\delta\varphi_s(0) + \delta\varphi_s(L)]$$
$$+\delta I_s^c(0) + \delta I_s^c(L), \qquad (57)$$

where $g_N = \sigma A/L$ (A being the area of the wire) is the conductance of the wire and $t(\lambda) = \tanh\lambda/\lambda$. In the limit $\lambda \to 0$, the conservation of the spin current is retained and $\Delta I_{s1}(0) = \Delta I_{s1}(L)$, as is seen in Eq. (57).

In the Boltzmann-Langevin formalism, the fluctuation of spin α currents through junction i are written in terms of the intrinsic current fluctuations $\delta I_{i\alpha}$ due to the random scattering of electrons from the tunnel barriers and the potential fluctuations $\delta\varphi_\alpha(0, L)$ at the junction points:

$$\Delta I_{i\alpha}(0, L) = \delta I_{i\alpha} - g_i\delta\varphi_\alpha(0, L). \qquad (58)$$

Using this relation the fluctuations of charge and spin currents through the terminals can be expressed in terms of the fluctuating spin and charge potentials at the connection points and the corresponding intrinsic currents fluctuations. Denoting $\delta I_{c(s)i}$ as the intrinsic fluctuations of the charge (spin) current through the tunnel junction i, we obtain

$$\Delta I_{c1} = \delta I_{c1} - g_1\delta\varphi_c(L) - g_1p_1\delta\varphi_s(L), \qquad (59)$$
$$\Delta I_{c2,3} = \delta I_{c2,3} - g_{2,3}\delta\varphi_c(0) - g_{2,3}p_{2,3}\delta\varphi_s(0), \qquad (60)$$
$$\Delta I_{s1}(L) = \delta I_{s1} - g_1p_1\delta\varphi_c(L) - g_1\delta\varphi_s(L), \qquad (61)$$
$$\Delta I_{s2,3} = \delta I_{s2,3} - g_{2,3}p_{2,3}\delta\varphi_c(0) - g_{2,3}\delta\varphi_s(0). \qquad (62)$$

Now we have to apply the currents conservation rules at the junction points. For spin-conserving tunnel barriers charge and spin currents fluctuations are conserved. At the pint $x = L$ the rules apply as equality of the expressions for ΔI_{c1} given in Eqs. (53) and (59), and $\Delta I_{s1}(L)$ in Eqs. (55) and (61). At

the point $x = 0$, they read

$$\sum_{i=1}^{3} \Delta I_{ci} = 0, \qquad (63)$$

$$\sum_{i=1}^{3} \Delta I_{si}(0) = 0, \qquad (64)$$

which in combination with Eqs. (53-57) and (59-62) lead to

$$\sum_i \delta I_{ci} = g_{23}\delta\varphi_c(0) + g_1\delta\varphi_c(L)$$
$$+g_{23}p_{23}\delta\varphi_s(0) + g_1p_1\delta\varphi_s(L) \qquad (65)$$
$$\sum_i \delta I_{si} = g_{23}p_{23}\delta\varphi_c(0) + g_1p_1\delta\varphi_c(L)$$
$$+(g_{23} + g_N t)\delta\varphi_s(0) + (g_1 + g_N t)\delta\varphi_s(L)$$
$$-\delta I_s^c(0) - \delta I_s^c(L) \qquad (66)$$
$$\delta I_{s1} = g_1p_1\delta\varphi_c(L) - \frac{g_N}{s}\delta\varphi_s(0)$$
$$+(g_1 + g_N\frac{\lambda^2}{t})\delta\varphi_s(L) - \delta I_s^c(L) \qquad (67)$$
$$\delta I_{c1} = -g_N\delta\varphi_c(0)$$
$$+(g_1 + g_N)\delta\varphi_c(L) + g_1p_1\delta\varphi_s(L) - \delta I_c^c \qquad (68)$$

The solution of this system of equations gives us the fluctuations of the potentials in the connecting nodes which can be replaced into Eqs. (59-62) to obtain the fluctuations of the charge and spin currents through different terminals in terms of δI_{ci}, δI_{si}, δI_c^c, and $\delta I_s^c(0, L)$. In particular, the fluctuations of the charge currents have the form

$$\Delta I_{ci} = \sum_{j=1}^{3} (c_{ij}\delta I_{cj} + c_{ij}^s\delta I_{sj})$$
$$+c_i\delta I_c^c + c_{i0}\delta I_s^c(0) + c_{iL}\delta I_s^c(L), \qquad (69)$$

where c_{ij}, c_{ij}^s, c_i, c_{i0}, and c_{iL} are functions of g_i, p_i, and λ.

B. Mean currents and correlations of currents fluctuations

The currents correlations $\langle\Delta I_{ci}\Delta I_{cj}\rangle$, are expressed in terms of the correlations of different fluctuating currents appearing in Eqs. (69). To calculate the correlations of the currents δI_c^c and $\delta I_s^c(0, L)$ we have to determine the mean distribution function $\bar{f}_{\alpha 0} = \bar{f}_{c0} + \alpha\bar{f}_{s0}$. This is achieved by solving Eqs. (24), (25) and imposing the boundary conditions that $\bar{f}_{\alpha 0}$ in the terminal F_i held in equilibrium at the voltage V_i is given by the Fermi-Dirac distribution function $f_i = F_{\text{FD}}(\varepsilon - eV_i)$. From the solutions of Eqs. (24), (25) we obtain

$$\bar{f}_\alpha = f_1 + (f_2 - f_1)[a + b\frac{x}{L}$$
$$+\alpha(c\sinh\frac{\lambda x}{L} + d\cosh\frac{\lambda x}{L})], \qquad (70)$$

where a, b, c, d are coefficients which have to be determined by the boundary conditions. Integrating of (70) over the energy ε the mean electro-chemical potential of spin α electrons

is obtained:

$$\bar{\varphi}_\alpha(x) = [a + b\frac{x}{L}$$
$$+\alpha(c\sinh\frac{\lambda x}{L} + d\cosh\frac{\lambda x}{L})]V, \tag{71}$$

which also could be obtained from the solutions of Eqs. (30), (35).

In the presence of the tunnel junctions the boundary conditions are imposed by applying the mean currents conservation rules at the connection points. Using Eqs. (28), (33), (71) we obtain the mean charge and spin currents through the N wire:

$$\bar{I}_{c1} = bg_N V, \tag{72}$$

$$\bar{I}_{s1}(x) = g_N\lambda(c\sinh\frac{\lambda x}{L} + d\cosh\frac{\lambda x}{L})V. \tag{73}$$

In terms of the charge and spin potentials at the connection points $\bar{\varphi}_c(0) = aV$, $\bar{\varphi}_c(L) = (a + bL)V$, $\bar{\varphi}_s(0) = dV$, $\bar{\varphi}_s(L) = (\lambda sc + \cosh\lambda d)V$, we have the following relations for the mean currents

$$\bar{I}_{c1} = -g_1\bar{\varphi}_c(L) - g_1 p_1\bar{\varphi}_s(L), \tag{74}$$
$$\bar{I}_{c2,3} = -g_{2,3}\bar{\varphi}_c(0) - g_{2,3}p_{2,3}\bar{\varphi}_s(0), \tag{75}$$
$$\bar{I}_{s1} = -g_1 p_1\bar{\varphi}_c(L) - g_1\bar{\varphi}_s(L), \tag{76}$$
$$\bar{I}_{s2,3} = -g_{2,3}p_{2,3}\bar{\varphi}_c(0) - g_{2,3}\bar{\varphi}_s(0),. \tag{77}$$

Using Eqs. (72-77) and the currents conservations relations at the point $x = 0$, $\sum_{i=1}^{3}\bar{I}_{ci} = 0$, and $\sum_{i=1}^{3}\bar{I}_{si}(0) = 0$, we find

$$\frac{a}{C} = [\frac{g_N}{g_{23}} + (1 + \frac{g_N}{g_1})q_{23} + \frac{g_1}{g_{23}}q_1]\cosh\lambda$$
$$+[\frac{g_N}{g_{23}}(1 + \frac{g_N}{g_1})\lambda^2 + (\frac{g_1}{g_N} + q_1)q_{23}]s$$
$$-p_1 p_{23}\frac{g_N}{g_{23}}, \tag{78}$$

$$\frac{1}{C} = [2\frac{g_N}{g_{23}} + (1 + \frac{g_N}{g_1})q_{23} + \frac{g_1}{g_{23}}(1 + \frac{g_N}{g_{23}})q_1]$$
$$\times\cosh\lambda + \frac{g_N}{g_{23}}(1 + \frac{g_N}{g_1} + \frac{g_N}{g_{23}})\lambda^2 s$$
$$+[q_1 + q_{23}(1 + \frac{g_1}{g_N}q_1)]s - 2p_1 p_{23}, \tag{79}$$

$$\frac{b}{C} = -(q_{23} + \frac{g_1}{g_{23}}q_1)\cosh\lambda$$
$$-[\frac{g_N}{g_{23}}\lambda^2 + \frac{g_1}{g_N}q_1 q_{23}]s, \tag{80}$$

$$\frac{c}{C} = -\frac{g_1}{g_{23}}(\frac{g_N}{g_1}\lambda^2 s + q_1\cosh\lambda)p_{23} - q_{23}p_1, \tag{81}$$

$$\frac{d}{C} = \frac{g_1}{g_{23}}\lambda(\frac{g_N}{g_1}\cosh\lambda + q_1 s)p_{23} - \frac{g_N}{g_{23}}\lambda p_1, \tag{82}$$

where $p_{23} = (g_2 p_2 + g_3 p_3)/g_{23}$, $g_{23} = g_2 + g_3$, $q_1 = 1 - p_1^2$, $q_{23} = 1 - p_{23}^2$, $s(\lambda) = \sinh\lambda/\lambda$.

Replacing $\bar{f}_{\alpha 0}$ given by Eq. (70) in Eqs. (47) we can calculate the correlations in Eqs. (42-46), which can be used to calculate all the possible correlations between δI_c^c and $\delta I_s^c(0, L)$,

given by Eqs. (54) and (56). Calculations lead to the following results

$$S_c = \langle\delta I_c^c\delta I_c^c\rangle = 2g_N[a(1 - a - b) + \frac{b}{2}(1 - \frac{2}{3}b)$$
$$+\frac{c^2 - d^2}{2} - s(cd\lambda s + \frac{c^2 + d^2}{2}\cosh\lambda)], \tag{83}$$

$$S_s(0) = \langle\delta I_s^c(0)\delta I_s^c(0)\rangle = 2g_N[\frac{a}{t}(1 - a)$$
$$+\frac{b}{2}(1 - 2a) + \frac{1}{2\lambda^2}(\frac{1}{s^2} - \frac{1}{t})b^2$$
$$+\frac{c^2 - d^2}{2s^2} - \frac{1}{s}(cd\lambda s + \frac{c^2 + d^2}{2}\cosh\lambda)], \tag{84}$$

$$S_s(L) = \langle\delta I_s^c(L)\delta I_s^c(L)\rangle = 2g_N[\frac{a}{t}(1 - a)$$
$$+(\frac{1}{t} - \frac{1}{2})b(1 - 2a) + (1 - \frac{1}{t} + \frac{1/s^2 - 1/t}{2\lambda^2})b^2$$
$$+\frac{c^2 - d^2}{2s^2} - \frac{1}{s}(cd\lambda s + \frac{c^2 + d^2}{2}\cosh\lambda)], \tag{85}$$

$$S_s(0L) = \langle\delta I_s^c(0)\delta I_s^c(L)\rangle = -2g_N[\frac{a}{t}(1 - a)$$
$$+\frac{b}{2s}(1 - 2a) + (\frac{1/t - 1}{2\lambda^2 s} - \frac{1}{2s})b^2 + \frac{c^2 - d^2}{2st}$$
$$-\frac{1}{t}(cd\lambda s + \frac{c^2 + d^2}{2}\cosh\lambda)], \tag{86}$$

$$S(0) = \langle\delta I_c^c\delta I_s^c(0)\rangle = g_N[(1 - 2a - 2b)d + (1 - 2a$$
$$-b)(\lambda c + \frac{\cosh\lambda}{s}d) + \frac{1}{\lambda}bc + (d - \frac{\cosh\lambda}{\lambda s}c)b], \tag{87}$$

$$S(L) = \langle\delta I_c^c\delta I_s^c(L)\rangle = -g_N[(1 - 2a - b)(\lambda sc$$
$$+\frac{1 + s\cosh\lambda}{s}d) - (\frac{\cosh\lambda}{s} - 1)(sd + \frac{\cosh\lambda}{\lambda}c)b]. \tag{88}$$

In writing these equations, we have for simplicity dropped the time dependence of the correlators and implicitly assumed, that the correlators are symmetrized (which leads to a factor of 2). Since we will be solely interested in the zero-frequency noise correlations, we also dropped the time integration.

To obtain the noise in the terminals we have to specify the correlators of the intrinsic fluctuations at the tunnel junctions. Assuming the tunnel junctions to be spin conserving, we obtain for the correlations of the intrinsic fluctuation of charge and spin currents

$$\langle\delta I_{ci}\delta I_{cj}\rangle = \langle\delta I_{si}\delta I_{sj}\rangle = \delta_{ij}2e\bar{I}_{ci} \tag{89}$$
$$\langle\delta I_{ci}\delta I_{sj}\rangle = \delta_{ij}2e(|\bar{I}_{ci+}| - |\bar{I}_{ci-}|). \tag{90}$$

where $\bar{I}_{ci\alpha} = \bar{I}_{ci} + \alpha\bar{I}_{si}$ is the mean current of spin α electrons.

Using the results (83-90) and Eqs. (59-62) with the solutions of Eqs. (65-68) the correlation of the currents of the form $S_{ij} = \langle\Delta I_{ci}\Delta I_{cj}\rangle$ is obtained. In terms of the coefficients introduced in (69) it has the form

$$S_{ij} = 2e\sum_{k=1}^{3}[(c_{ik}c_{jk} + c_{ik}^s c_{jk}^s)\bar{I}_k + (c_{ik}c_{jk}^s$$
$$+c_{ik}^s c_{jk})(|\bar{I}_{c+k}| - |\bar{I}_{c-k}|)] + c_i c_j S_c + c_{i0}c_{j0}S_s(0)$$
$$+c_{iL}c_{jL}S_s(L) + (c_i c_{j0} + c_{i0}c_j)S(0) + (c_i c_{jL}$$
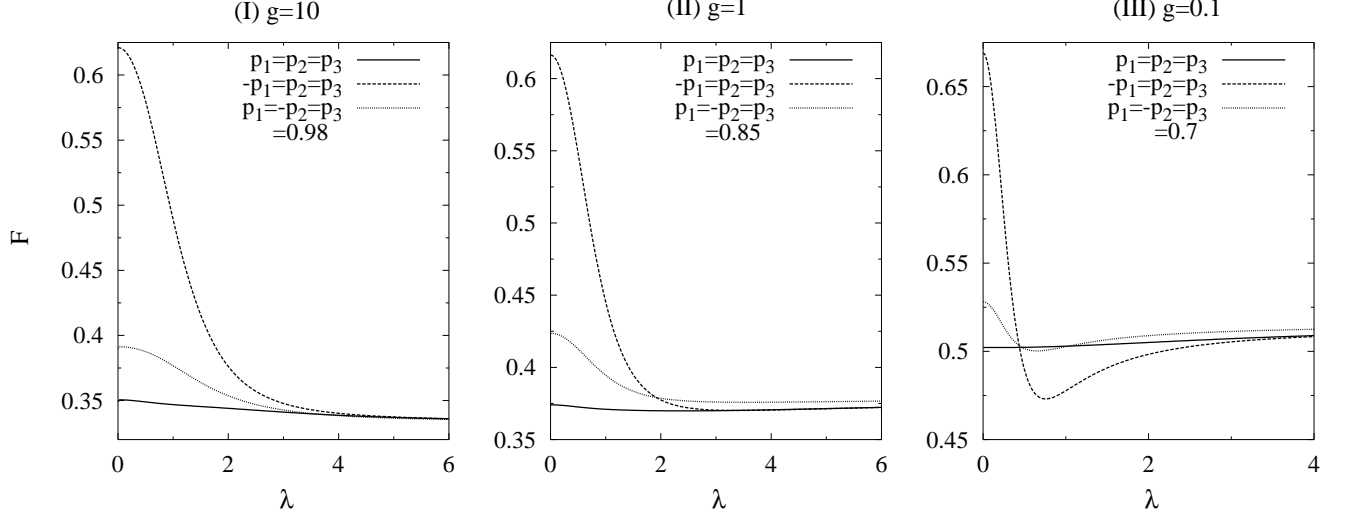$$+c_{iL}c_j)S(L) + (c_{i0}c_{jL} + c_{iL}c_{j0})S_s(0L). \tag{91}$$

FIG. 2: Fano factor F versus the spin flip scattering intensity $\lambda = L/\ell_{sf}$ for a given magnitude of the polarization in F-terminals but different configurations and for different values of the tunnel conductances $g = g_i/g_N$. For the configurations with $p_2 = p_3$ the corresponding cross-correlation is obtained via $S_{23}/|I_{c1}| = (F-1)/4$.

In this way we obtain the Fano factor $F = S_{11}/2e|I_{c1}|$ and the cross correlations S_{23} measured between the currents through F_2, F_3. In the general case for arbitrary g_i, p_i and λ the expressions of F and S_{23} are too lengthy to be given here and in the next section we will present analytical expressions of F and S_{23} in some important limits only.

IV. RESULTS AND DISCUSSION

For simplicity in the following we will consider the junctions to have the same tunneling conductances $g_i/g_N = g$, $(i = 1, 2, 3)$. If the amplitude of the polarizations $\{p_i\}$ are also the same we distinguish the following different configurations. The two terminals F_2 and F_3 have either parallel or anti-parallel polarizations. We take the signs of p_2 and p_3 to be positive in the parallel configuration. In this case there are two different configurations depending on whether p_1 is positive (parallel to p_2, p_3) or negative (anti-parallel to p_2, p_3). On the other hand for the antiparallel configuration of F_2 and F_3 the sign of p_1 is not essential and the two cases of p_1 and $-p_1$ are equivalent. Thus there are three independent configurations of the polarizations corresponding to $p_1 = p_2 = p_3 = p$, $-p_1 = p_2 = p_3 = p$ and $p_1 = -p_2 = p_3 = p$. We denote these configurations by "$+$", "$-$", and "0", respectively.

We analyze the dependence of F and $F_{23} = S_{23}/|I_{c1}|$ on the spin flip scattering intensity λ for different configurations and amplitude of the polarizations. We show how the spin flip scattering affects both F and S_{23} and produces a strong dependence on the magnetic configuration of the F terminals.

A. Shot noise

Let us start with analyzing of the shot noise. Fig. 2 illustrates the typical behaviour of the shot noise with respect to the spin flip scattering intensity and configuration of the polarizations. Here F versus λ is plotted for a given magnitude of the polarization p in the terminals for the different magnetization configurations. Different columns I-III belong to different values of the tunnel contact conductances g. Clearly, for a finite λ the Fano factor F is changes drastically with the relative orientation of the polarizations. For strong spin flip scattering, $\lambda \gg 1$ the Fano factor reduces to

$$F_N = \frac{1}{3}\frac{5 + 6g + 4g^2 + (8/9)g^3}{3 + 6g + 4g^2 + (8/9)g^3}, \tag{92}$$

independent of the polarization of the terminals. Eq. (92) is the result for a all-normal metal three terminal system $(p_1 = p_2 = p_3 = 0)$[9] which reduces to $5/9$ and $1/3$ in the limits of small and large g, respectively. This is expected since the strong spin flip intensity destroys the polarization of the injected electrons.

At finite λ the curves belonging to different configurations differ from each other and the largest difference occurs as λ approaches zero. In this limit, using Eqs. (91) and (74) we obtain the following results for the different configurations

$$F_+ = \frac{45G_+^2}{g^2x_+^3}[q(\frac{4-2q}{9}g^2 + g + \frac{3}{2})g + \frac{3}{4}]$$
$$+ \frac{16G_+^3}{3x_+^4g^2}\{q^3(qg + \frac{21}{2})g^5 + \frac{3}{2}q[q(22 + 7q)g + 36 + \frac{51}{2}q]g^3$$
$$- \frac{81}{32}q[(6q - 32)g^2 + 11g + 6] + \frac{243}{8}(\frac{4}{3}g^2 + 3g + 1)\}, \tag{93}$$

$$F_- = \frac{G_-^2}{q^2 g^2 x_-^3}[10q^3(2-q)g^3 + 9q^2(7-2q)g^2 + 6q(\frac{5}{4}q^2$$

$$-6q+16)g + 3(\frac{13}{4}q^2 - 8q + 16)] + \frac{8G_-^3}{qx_-^4 g^2}[\frac{2}{3}q^5 g^6$$

$$+7q^4 g^5 + q^3(3q+26)g^4 + \frac{3}{2}q^2(9q+32)g^3$$

$$+\frac{1}{3}q(-\frac{83}{8}q^2 + 86q + 137)g^2 + 9(-\frac{17}{16}q^2 + \frac{15}{4}q + 2)g$$

$$+\frac{q^2}{8} - 7q + 17], \tag{94}$$

$$F_0 = \frac{2G_0^2}{g^2 x_0^3}[q(q^2 - 4q + 8)g^3 + 3q(\frac{q^2}{2} - q + 8)g^2$$

$$+3q(\frac{q^2}{4} - q + 12)g + \frac{q^3}{8} - \frac{5}{4}q^2 + 10q$$

$$+8] + \frac{8G_0^3}{x_0^4 g^2}[\frac{2}{3}q^4 g^6 + \frac{7}{3}q^3(q+2)g^5 + q^2(3q^2$$

$$+18q+8)g^4 + q(\frac{11}{6}q^3 + 25q^2 + 24q + \frac{32}{3})g^3 + \frac{1}{3}(\frac{13q^4}{8}$$

$$+55q^3 + 44q^2 + 96q + 16)g^2 + (\frac{q^4}{16} + \frac{57}{8}q^3 + q^2$$

$$+\frac{57q}{4} + 16)g + \frac{9}{8}q^3 - q^2 + 2q + 8]. \tag{95}$$

Here we defined $x_\pm = 2qg + 3$, $x_0 = 2qg + q + 2$, and $q = 1 - p^2$. The total conductances of the system normalized by g_N for the three configurations in the limit $\lambda \to 0$ are given by

$$G_+ = \frac{gx_+}{2qg^2 + 6g + 9/2},$$

$$G_- = \frac{2qgx_-}{4q^2 g^2 + 12qg + q + 8},$$

$$G_0 = \frac{gx_0}{2qg^2 + 2qg + 4g + q/2 + 4}.$$

In Fig. 3 we show the polarization dependence of the Fano factor in the limit of small spin-flip scattering intensity, $\lambda \to 0$ for the different magnetic configurations of the terminals. At $p = 0$ the Fano factor for different conductances g takes the normal state value F_N, see Eq. (92), independent of the polarizations configuration. For finite polarizations F of different configurations differ from each other and the normal state value. As p increases, the Fano factor of the configuration $-$ deviates substantially from those of the other two configurations and reaches the full Poissonian value 1 as p approaches 1. This is independent of the tunnel conductances g. Thus for perfectly polarized terminals the Fano factor takes the value 1 in the limit of small spin-flip scattering rate.

To understand this effect we note that in the limit $p \to 1$, the system constitutes an ideal three terminal spin valve due to the antiparallel configuration of the polarizations at its two ends. In the absence of the spin flip scattering there in no current through the N-wire since for the up-spin electrons provided by the terminals F_2 and F_3, there is no empty state in the terminal F_1 in the energy range eV. For very small but finite
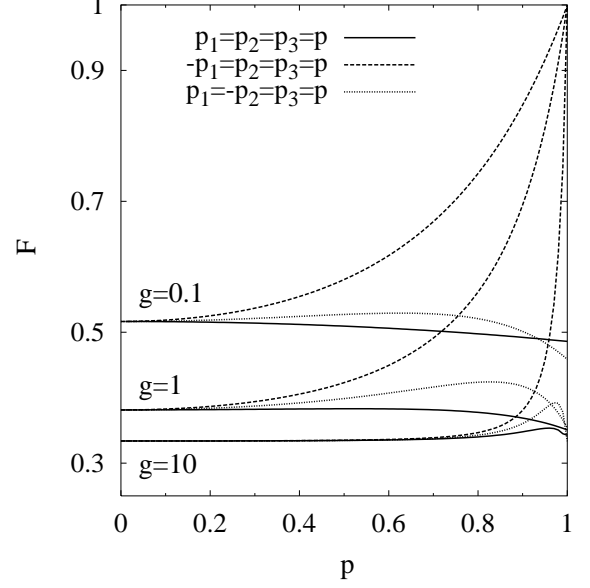

FIG. 3: Fano factor F versus the magnitude of spin polarization p of the terminals in the limit of small spin flip-scattering intensity $\lambda = L/\ell_{sf} \to 0$. The results are shown for different configurations of the polarizations and different values of the tunnel conductances g.

λ only those of electrons which undergo spin-flip scattering once can carry a small amount of current. These spin-flipped electrons are almost uncorrelated and pass through the normal wire independently giving rise to full Poissonian shot noise. Similar effects have been found before for two[14] and four[18] terminal spin-valve systems.

For arbitrary g and p, F has a complicated dependence on λ and the corresponding expressions are to lengthy to be presented here. Simpler expressions are obtained for perfectly polarized junctions. Setting $p = 1$ in Eqs. (91) and (74) yield for the Fano factors of the different configurations

$$F_+ = \frac{5G_+^2}{g^2}$$

$$+\frac{8G_+^3}{g^2}[\frac{1}{3}g^2 + \frac{3}{2}g + 1 + 2(g^2 + \frac{2g^2}{\lambda^2} + 3g + 1)\frac{\tanh(\lambda/2)}{\lambda}$$

$$+4g(g\frac{\cosh^2 \lambda - 3s}{\lambda^2 s^2} + \frac{3}{2})\frac{\tanh^2(\lambda/2)}{\lambda^2}], \tag{96}$$

$$F_- = \frac{5G_-^2}{g^2}$$

$$+\frac{8G_-^3}{g^2}[\frac{1}{3}g^2 + \frac{3}{2}g + 1 + 2(g^2 + \frac{+2g^2}{\lambda^2} + 3g + 1)\frac{\coth(\lambda/2)}{\lambda}$$

$$+4g(g\frac{\cosh^2 \lambda + 3s}{\lambda^2 s^2} + \frac{3}{2})\frac{\coth^2(\lambda/2)}{\lambda^2}], \tag{97}$$

$$F_0 = \frac{G_0^2}{g^2}[g^2(1+\frac{1}{\lambda^2})+3g+\frac{9}{4}-\frac{3G_0}{g}(\frac{2}{9}g^3+g^2+\frac{3}{2}g$$

$$+\frac{1}{2})]-\frac{G_0}{x^2}[4g^2+2g-\lambda^2+\frac{8G_0}{g}(g^3(\frac{1}{\lambda^2}-1)-2g^2$$

$$+g(\frac{1}{4}\lambda^2-1)+\frac{3}{8}\lambda^2)+\frac{4G_0^2}{g^2}(g^4(1-\frac{2}{\lambda^2})+g^3(\frac{7}{2}-\frac{2}{\lambda^2})$$

$$+\frac{g^2}{4}(17-\lambda^2)+g(1-\frac{3}{4}\lambda^2)-\frac{9}{16}\lambda^2)]. \tag{98}$$

Here we defined $x = 2g\cosh\lambda + \lambda^2 s(\lambda)$ and the dimensionless total conductances are now given by

$$G_+ = \frac{g}{2g+4g\tanh(\lambda/2)/\lambda+3},$$

$$G_- = \frac{1}{2g+4g\coth(\lambda/2)/\lambda+3},$$

$$G_0 = \frac{gx}{gx+4g\cosh\lambda+(2g^2+3\lambda^2/2)s(\lambda)}.$$

The strong dependence of F on λ, and the magnetic configuration is also shown in Fig. 4, where F versus λ is plotted for different polarization p_1 of the terminal F_1, and fixed $p_2 = p_3 = 1$. As in Fig. 2 different columns I-III present results for different values of g. In each column p_1 varies from top to bottom in the interval -1 to 1. In the limit of large λ, the Fano factor tends to the the normal state value (92) determined by the conductance g only. The deviations from this normal state value at finite λ depend on p_1 and g.

For small values of g (column III), with decreasing λ from large values F first decreases below the normal state value F_N and then starts to increase again. Thus, there is a minimum of the Fano factor F occurring at a value of λ which continuously decreases from $\lambda \sim 1$ to $\lambda = 0$ as p_1 increases from -1 to 1. Decreasing λ further, F increases to a maximum value at $\lambda = 0$. The maximum of F at $\lambda = 0$ continuously decreases with increasing p_1 and becomes a minimum when $p_1 = 1$. For antiparallel fully polarized terminals, i. e. $p_1 = -1$, decreasing λ leads to the strongest variation of F, see Eq. (97) and F reaches the Poissonian value 1 as λ approaches zero. Comparing plots for $p_1 = -1$ of columns (I-III), we see that this effect is independent of the contact conductances g, which is in agreement with the discussion in connection with Fig. 3. Increasing p_1 from -1 decreases the effect of spin flip process in the current and the noise, and, hence, leads to a reduction of the shot noise at $\lambda \ll 1$. The maximum value of F thus drops below the Poissonian value.

For large values of g (column I) the maximum of F is shifted from zero to a finite $\lambda \sim 1$ as p_1 increases from -1 to 1, while for small g the maximum F always occurs at $\lambda = 0$ as described above. For $g \sim 1$ (column II) the situation is in between the large and small g behavior, increasing λ leads first to a maximum of the Fano factor followed by a minimum. Comparing the first and the second row of Fig. 4, we observe the effect of spin-flip scattering is more pronounced for negative p_1 than for positive p_1. This can be understood, because for parallel magnetization directions the non-equilibrium spin accumulation in the ferromagnetic wire is smaller. The spin-flip scattering decreases the spin-accumulation and, hence, has the largest effect for anti-parallel magnetizations.

B. Cross-correlations

Let us now discuss the effect of spin flip scattering on the cross correlations measured between the currents through the terminals F_2 and F_3. We distinguish two different cases of parallel ($p_2 = p_3$) and antiparallel ($p_2 = -p_3$) magnetizations of F_2 and F_3. For the parallel case and when $g_2 = g_3$ the two terminals F_2, F_3 are completely equivalent and hence $\Delta I_2 = \Delta I_3$. This can be used with Eq. (63) to obtain that in this case the Fano factor and the cross-correlation factor $F_{23} = S_{23}/|2eI_{c1}|$ are related as (see also Ref.[25])

$$F_{23} = \frac{F-1}{4}. \tag{99}$$

Thus F_{23} has the same qualitative dependence on λ as F. Since $F \leq 1$ the cross-correlations are always negative as expected[31]. The Fano factor for the perfectly polarized parallel case is presented in Fig. 4 and the cross-correlations for this case can be deduced from these plots using Eq. (99). We will now analyze the cross-correlations F_{23} for $p_2 = p_3 = 1$ and different values of p_1. From Eqs. (91) and (74) one can see that in the limit of large spin-flip scattering $\lambda \gg 1$, the cross-correlations reduces to its all-normal system value

$$F_{23} = -\frac{1}{9}\frac{(4/3)g^3+6g^2+9g+3}{(8/9)g^3+4g^2+6g+3}, \tag{100}$$

which is independent of the polarizations. Alternatively, this result could have been obtained using Eqs. (92) and (99). On lowering λ the amplitude of the cross-correlations $|F_{23}|$ decreases (large g, column I) or increases (small g, column III) with respect to the normal value. For $p_1 = -1$, F_{23} vanishes in the limit $\lambda \to 0$, irrespective of the the value of the contacts conductance g. This case corresponds to the vanishing of the total mean current $\bar{I}_1$. From this observation we conclude that in the expression of F_{23}, the cross-correlation S_{23} vanishes faster than the mean current $\bar{I}_1$, as $\lambda \to 0$. Thus, for $p_1 = -1$ $|F_{23}|$ has a minimum at $\lambda = 0$ for both cases of small and large g. Increasing p_1 from -1, the minimum is shifted to a finite $\lambda \sim 1$ for large g, it stays always at $\lambda = 0$ for small g. For small g, $|F_{23}|$ has also a maximum at $\lambda \sim 1$ which corresponds to the minimum of the Fano factor.

For $g \sim 1$ (column II) the behavior of F_{23} is between that for small and large g. Comparing of the plots in top ($p_1 < 0$) and down ($p_1 > 0$) rows in Fig. 4 shows that the most strong variation of the cross-correlation happens when the magnetization of F_1 is anti-parallel to those of $F_{2,3}$.

For the antiparallel case $p_2 = -p_3$ the effect of spin-flip scattering is more interesting, since it contains the correlations between currents of opposite spin directions produced by the spin-flip scattering. In Fig. 5 we plot F_{23} versus λ for different values of the magnitude $|p_2| = |p_3| = p$ and g. We take $p_1 = 1$ which corresponds to a maximum spin accumulation in the N-wire due to the terminal F_1. For $p = 1$ the cross-correlations are solely due to the spin flip-scattering. In this case F_{23} vanishes in the limit of $\lambda \to 0$.

At finite λ the spin-flip scattering induces correlations between the electrons with opposite spins and hence F_{23} be-
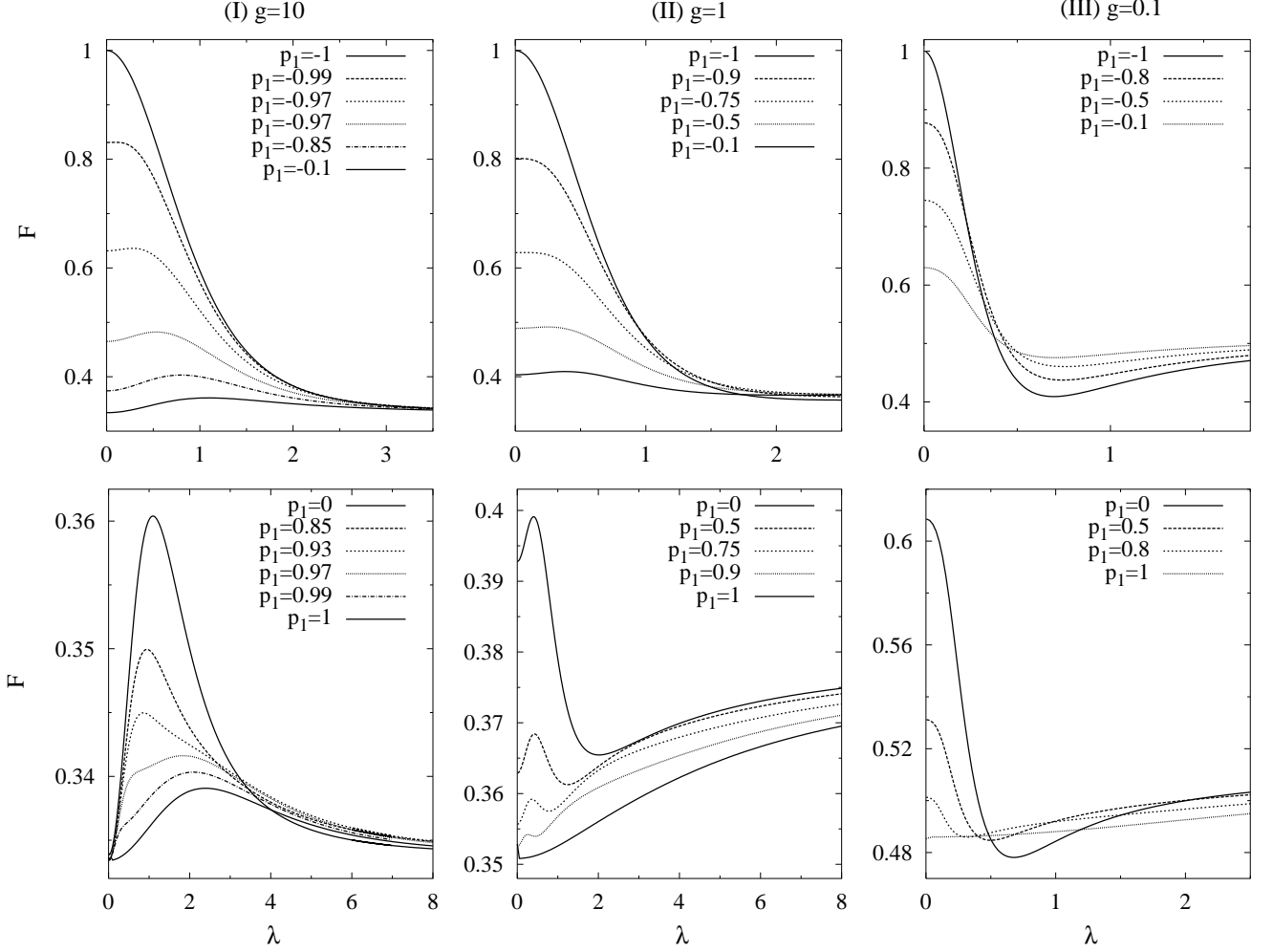
FIG. 4: Fano factor F versus the spin flip scattering intensity $\lambda = L/\ell_{sf}$ for different polarizations p_1 in the terminal F_1, and perfect polarizations in the other two terminals F_2, F_3: $p_2 = p_3 = 1$. The columns I-III correspond to different values of the tunnel contact conductances $g = g_i/g_N$. Here the cross-correlations are related to the Fano factor via $S_{23}/|I_{c1}| = (F-1)/4$. For explanation of the various plots, see text.

comes finite. With increasing λ the magnitude of F_{23} increases and approaches the all-normal system value of $p_1 = p_2 = p_3 = 0$ when $\lambda \gg 1$. For $p < 1$ the value of $|F_{23}|$ for vanishing λ depends on the values of p and g as

$$F_{23} = -\frac{9}{8}(1-p^2)\frac{(4/3)g^3 + 4g^2 + 4g + 1}{(g+1)^3}. \quad (101)$$

Decreasing p from 1 to 0, $|F_{23}|$ increases from zero to a maximum value. The maximum absolute value is equal to the normal value for large g (column I), while it is larger than the normal value for small g (column III).

V. CONCLUSIONS

In conclusion, we have investigated the influence of spin polarization and spin-flip scattering on current fluctuations in a three-terminal spin-valve system. Based on a spin-dependent Boltzmann-Langevin formalism, which accounts for spin-flip scattering in addition to the usual scattering at impurities and tunnel junctions, we have developed a semiclassical theory of current fluctuations in diffusive spin-valves. This theory allows the calculations of spin-polarized mean currents and correlations of the corresponding fluctuations in multi-terminal systems of diffusive wires and tunnel contacts.

We have applied this formalism to a three-terminal system consisting of a diffusive normal wire connected at the ends to one and two ferromagnetic terminals, respectively. We have found a strong deviation of the current correlations from the all-normal system values. The shot noise of the total current through the system and the cross-correlations between currents of two different terminals depend strongly on the spin-flip scattering rate and the spin polarization and change drastically with reversing the polarizations in one or more of the terminals.

The strongest variation of the shot noise occurs, when the polarizations of the two terminals connected to one end of the normal wire are antiparallel with respect to the terminal on the
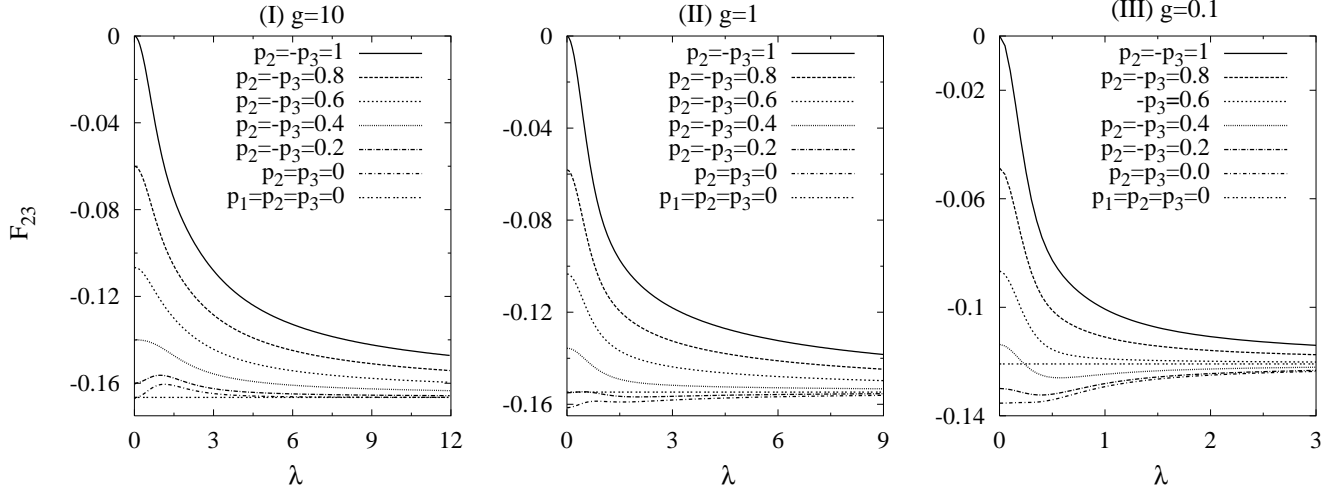
FIG. 5: Cross-correlations of the current measured between terminals F_2 and F_3 for anti-parallel configuration of the polarizations $p_2 = -p_3$ and for different contact conductances $g = g_i/g_N$. The polarization of F_1, $p_1 = 1$ and the magnitude of the magnetizations $|p_2| = |p_3|$ vary in each column.

other end. For small spin-flip scattering intensity, the Fano factor deviates substantially from the normal value and can reach the full Poissonian value for perfectly polarized terminals even if the tunnel contact resistances are negligible.

We have further demonstrated the effect of spin-polarization and spin-flip scattering on the cross-correlations measured between currents of two adjacent terminals in two cases where the terminals have parallel and antiparallel polarizations. For antiparallel orientations of the contact polarizations, the noise allows a direct determination of the spin-flip scattering processes in the normal wire. The study of noise and cross-correlations therefore allows to extract information

on the spin-flip scattering strength, which is of importance for spintronics applications.

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